भारतीय वन सेवा परीक्षा 2011

Indian Forest Services Examination

10241

D-VSF-L-CUA

PHYSICS

Paper I

Time Allowed : Three Hours | Maximum Marks : 200

INSTRUCTIONS

Candidates should attempt Questions No. 1 and 5 which are compulsory, and any THREE of the remaining questions, selecting at least ONE question from each Section.

All questions carry equal marks.

The number of marks carried by each part of a question is indicated against each.

Answers must be written in ENGLISH only.

Assume suitable data, if necessary, and indicate the same clearly.

Unless otherwise indicated, symbolic notations carry usual meaning.

Useful Constants :

Electron charge (e) = $1{\cdot}602 \times 10^{-19}$ C

Electron rest mass (m_e) = $9{\cdot}109 \times 10^{-31}$ kg

Proton mass (m_p) = $1{\cdot}672 \times 10^{-27}$ kg

Vacuum permittivity (ε_0) = $8{\cdot}854 \times 10^{-12}$ farad/m

Vacuum permeability (μ_0) = $1{\cdot}257 \times 10^{-6}$ henry/m

Velocity of light in free space (c) = 3×10^8 m/s

Boltzmann constant (k) = $1{\cdot}38 \times 10^{-23}$ J/K

Electron volt (eV) = $1{\cdot}602 \times 10^{-19}$ J

Planck's constant (h) = $6{\cdot}62 \times 10^{-34}$ J-s

Stefan's constant (σ) = $5{\cdot}67 \times 10^{-8}$ W m^{-2} K^{-4}

Avogadro's number (N) = $6{\cdot}02 \times 10^{26}$ $kmol^{-1}$

Gas constant (R) = $8{\cdot}31 \times 10^3$ J $kmol^{-1}$ K^{-1}

exp (1) = 2·7183

SECTION A

1. Answer any ***four*** of the following : *4×10=40*

(a) A particle of mass 'm' moves according to the equations x = a cos ωt, y = a sin ωt, z = c, where a, c, ω are constants. Obtain the instantaneous velocity and linear momentum vectors in terms of the Cartesian components and hence the angular momentum. Find the force $\vec{F}$ and the torque $\vec{N}$ acting on the particle and verify that the angular momentum $\vec{L}$ and the torque $\vec{N}$ satisfy the relation $\frac{d\vec{L}}{dt} = \vec{N}$.

(b) A particle of rest mass 'M' moving at a velocity 'u' collides with a stationary particle of rest mass 'm'. If the particles stick together, show that the speed of the composite ball is equal to

$$v = \frac{u\gamma M}{(\gamma M + m)}, \text{ where } \gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}.$$

(c) A plano-convex lens of crown glass is connected to a concavo-convex lens of flint glass so that the convex surface of the first and the concave surface of the second fit exactly. If the combination is achromatic and the combined focal length is -50 cm, determine the radii of curvature of the faces of the flint glass lens. The dispersive power of the crown glass is one-half of that of the flint glass and the refractive indices for yellow light are respectively 1·500 and 1·625 respectively.

(d) A diffraction grating has 5000 lines per cm. For illumination at normal incidence, determine the dispersive power of the grating in the second order spectrum in the range of wavelengths around 500 nm.

(e) Consider a Gaussian pulse propagating in pure silica, at $\lambda_0 = 0{\cdot}85\ \mu m$, along z-direction. As the pulse propagates, it gets broadened according to the formula

$$\tau^2(z) = \tau_0 \left(1 + \frac{4\alpha^2 z^2}{\tau_0^4}\right), \alpha = \frac{d^2k}{d\omega^2}\bigg|_{\omega=\omega_0}$$

where $\tau(z)$ is the pulse width after propagation through a distance z and τ_0 is the pulse width at $z = 0$. Given that $\alpha = 3 \times 10^{-26}\ \frac{S^2}{m}$ at $\lambda_0 = 0{\cdot}85\ \mu m$. Calculate the distance at which $\tau(z) = \sqrt{2}\cdot \tau_0$.

2. (a) A rigid body rotates with the angular velocity 'ω' about an axis through the origin O and having direction cosines l, m, n. Show that the moment of inertia of the rigid body about the axis is

$$I = I_{xx}l^2 + I_{yy}m^2 + I_{zz}n^2 + 2\,I_{xy}l.m + 2\,I_{yz}m.n + 2\,I_{zx}n.l$$

where the symbols have their usual meanings. *20*

(b) The principal moments of inertia of a body at a point are given as 200, 300 and 450 gm.cm^2. Write down the equation of the ellipsoid of inertia at that point. *10*

(c) A light spring of relaxed length 'a_0' is suspended from a point. It carries a mass 'm' at its lower free end, which stretches it through a distance l. Show that the vertical oscillation of the system is simple harmonic in nature and has time period $T = 2\pi\sqrt{l/g}$, where g is the acceleration due to gravity. *10*

3. (a) In the steady state forced vibration of the damped harmonic oscillator, show that the amplitude of the driven system is maximum when $p = \sqrt{\omega^2 - 2b^2}$ and further the value of the maximum amplitude is $\frac{f}{2b\sqrt{\omega^2 - b^2}}$, where the symbols have their usual significance. (Here you start from the steady state solution) *5*

(b) In a double slit experiment the slits are 2 mm apart and are illuminated with a mixture of two wavelengths, $\lambda = 750$ nm and $\lambda' = 900$ nm. At what minimum distance from the common central bright fringe on a screen 2 m distant from the slits will a bright fringe from one interference pattern coincide with a bright fringe from the other ? *10*

(c) What is Fermat's principle ? Derive the laws of reflection and refraction with the help of this principle, when the light is incident on a curved surface. *25*

4. (a) Explain the diffraction at straight edge with the help of Cornu's spiral. *25*

(b) Distinguish between the intensity patterns due to diffraction from a narrow slit and a straight edge. *15*

SECTION B

5. Answer any ***four*** questions from the following : *4×10=40*

(a) The electric field in a medium is given by

$$\vec{E} = \vec{E}_0 e^{-\alpha z} \sin(kz - \omega t),$$

where $\vec{E}_0$ is a constant vector with dimensions of the electric field. Prove that $\vec{E}$ cannot have a component along the unit vector, $\hat{z}$, parallel to the z-axis. Here α is a positive constant.

(b) Show that the elemental quantity of heat δQ is not a total differential.

(c) Prove that the Maxwell's equations in a medium contain the conservation of charge in differential form.

(d) An inductor of inductance 5 H is suddenly connected to a 10 V d.c. power supply through a resistor of 10 Ω. After what time will the current in the circuit be 1/10th of its steady state value ?

(e) Consider the Earth as a black body. Radiations from the Sun arrive at the surface of the Earth with an average intensity of S watts/m^2. If the reflection coefficient of the Earth's surface is α, determine the temperature of the Earth under equilibrium conditions.

6. (a) Write down the expression for the Bose – Einstein distribution function and explain the meaning of the symbols used. 10

(b) Consider a gas of photons in equilibrium and contained in a volume V at a temperature T. Using the Bose – Einstein statistics, calculate the total energy of the photon gas. 20

(c) If the magnetic field $\vec{B}$, at a point with position vector $\vec{r}$ is uniform, show that the corresponding vector potential $\vec{A}(\vec{r})$ is given by

$$\vec{A}(\vec{r}) = -\frac{1}{2}\left[\vec{r} \times \vec{B}\right].$$

10

7. (a) Consider an infinite current sheet with a uniform current density $\vec{K}$ (Amp/m). Show that the magnetic field $\vec{H}$ at a point away from the sheet is

$$\vec{H} = \frac{1}{2}\vec{K} \times \hat{n}$$

where $\hat{n}$ is a unit normal vector directed from the current sheet to the point. 25

(b) The plane y = 5 carries a current of density $10\ \hat{z}$ (Amp/m). Calculate the value of the magnetic field $\vec{H}$ at the point (0, 1, –5). 15

8. (a) Consider an infinite line charge with charge density ρ coulomb/meter located at a distance d meters from a grounded conducting plane z = 0. Determine :

   (i) the magnitude of the potential V for $z > 0$ and $z \leq 0$. *15*

   (ii) the surface charge density induced on the conducting plane. *10*

(b) The current density in spherical co-ordinates is given by

$$\vec{J} = \frac{1}{r^3}\left[2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right] \text{ A/m}^2$$

where $\hat{r}$ and $\hat{\theta}$ are unit vectors. Calculate the amount of current passing through a hemisphere of radius 20 cm. *15*